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# On the importance of aerodynamic and structural geometrical nonlinearities in aeroelastic behavior of high-aspect-ratio wings ☆

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**Abstract**

Theoretical development of a nonlinear aeroelastic analysis for high-aspect-ratio wings is presented. The analysis couples a geometrically exact beam theory with a nonplanar aerodynamic theory. The aim of the present effort is to investigate the effects of geometrical nonlinearities on the aeroelastic behavior of high-aspect-ratio wings. The focus of the present work is on characterizing the physical origins of nonlinearities and assessing their relative importance. Results presented show that the structural geometrical nonlinearities have a significant effect on the structural dynamics and on the dynamic aeroelastic characteristics of a high-aspect-ratio wing. The geometrically exact calculation of the angle of attack and aerodynamically consistent application of the airloads is also important for accurate aeroelastic characterization. On the other hand, the geometrical aerodynamic nonlinearity emerging from the nonplanar wake effects is quite negligible. Nonplanar effects were negligible for both the steady and unsteady airload calculations with the assumption of a fixed wake model.


## 1. Introduction

There has been a growing interest in high-altitude, long-endurance (HALE) aircraft in recent years. These aircraft are being considered for unmanned reconnaissance missions, long-term surveillance, environmental sensing, and communications relay. HALE aircraft have slender wings (aspect ratio of the order of 35), which are highly flexible. Due to the high flexibility and large aspect ratios, large deflections can result, reaching about 25% of wing semi-span. Linear theory fails to accurately analyze such deformation and the changes in the structural and aerodynamic characteristics of the wing accompanying such deformation.

Aeroelastic characteristics of highly flexible aircraft have been investigated by van Schoor and von Flotow (1990). The complete aircraft was modelled using a few modes of vibration, including rigid-body modes. Linear aeroelastic and flight dynamic analysis results for a HALE aircraft has been presented by Pendaries (1998). The results highlight the effects of wing flexibility on the aeroelastic characteristics of the wing and the flight dynamic characteristics of the

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aircraft. Both analyses, though focused on the aeroelastic characteristics of high-aspect-ratio wings, are linear and do not take into account the geometrical nonlinearities induced by large deflections.

Patil et al. (1999) have looked at the effect of structural geometric nonlinearities on the flutter behavior of high-aspect-ratio wings. A steady state deflection of the wing is calculated based on constant distributed loading and the changes in structural and aeroelastic characteristics are presented. The results indicate a significant change in the structural frequencies and a significant reduction in the flutter speed. A two-dimensional (2-D) aerodynamic model was used and thus the aerodynamic nonlinearities (due to curvature) are not present. Theoretical and experimental investigation of flutter and limit cycle oscillations using a nonlinear beam model and ONERA stall model has been conducted by Tang and Dowell (2001). On the other hand, Hall et al. (1999) presents the results obtained by using a three-dimensional (3-D) geometrically exact (nonplanar) aerodynamic theory coupled with a linear structural analysis. Again the flutter instability speed was drastically reduced with wing curvature. The results were based on free-wake aerodynamic analysis.

The goal of the present work is to investigate the importance of analyzing the structure as well as aerodynamics in a geometrically exact manner. It will be shown that inclusion of the geometric nonlinearities is required for accurate aeroelastic predictions, even under normal flight conditions. In a nutshell, the wings of HALE aircraft are beam-like and highly curved during flight, and curved beams behave very differently from straight beams. The aeroelastic characteristics of a curved wing are affected by curvature-induced changes in the effective bending-torsion coupling and the changes in unsteady aerodynamic loading (due to change in the location of the bound and shed vortices). It is necessary to study the relative importance of these effects in predicting the aeroelastic characteristics of a flexible, curved high-aspect-ratio wing.

### 1.1. Geometric nonlinearities

Nonlinearities enter the system through various physical mechanisms. The geometric nonlinearities that are included in the present theoretical development are manifested through the structural formulation, the aerodynamic formulation, and the fluid–structure interface. Before investigating the relative importance of the various nonlinearities, it is helpful to understand the physical mechanisms responsible for the various nonlinearities.

Structural geometric nonlinearities can be attributed to nontrivial steady state deformation and/or large motion. The basis of the nonlinearity lies in kinematics, particularly the relations expressing generalized velocity and strain measures in terms of displacement and rotation variables of the wing.

Aerodynamic geometrical nonlinearities are manifested in a 3-D aerodynamic model because of the dependence of the pressure at a given point on the disturbances throughout the surface. Using linear aerodynamic theory, the pressure at a point can be linearly related to displacement at another point in terms of an influence coefficient. But since the influence coefficient itself is a function of the wing geometry, the pressure becomes a nonlinear function of the displacements.

Finally, the complete aeroelastic model will be geometrically exact only if the fluid–structure interface is handled so as to be consistent with both structure and fluid models. The downwash distribution required by the aerodynamic model is determined by the structural displacements, whereas the forces on the structure are calculated by the aerodynamic loads. The word "consistent" implies the geometrically correct transfer of angle-of-attack information from the structural model to the aerodynamic model and the geometrically correct transfer of airloads from the aerodynamic model to the structural model. The downwash must be defined as the component of the relative velocity of the fluid with respect to the structure, which is perpendicular to the surface. On the other hand, the forces are calculated in terms of a cross-product of the vortex (or doublet) and the free-stream velocity. Thus, the computation of the downwash and the application of the aerodynamic forces has to be accomplished using a consistent vectorial approach.

### 1.2. Procedure for investigation

The procedure for investigation of the geometric nonlinearities is based on the calculation of various nonlinear steady state solutions of the aeroelastic system and linearizing the problem about the steady state solutions. The nonlinear steady state solution is helpful in identifying the static nonlinearities, whereas the dynamic linear perturbation of the problem is helpful in identifying the changes in the dynamic aspects of the aeroelastic problem. The third component of nonlinear aeroelastic analysis involves large-amplitude motion (Patil et al., 2001b). This component is outside the scope of the present investigation.

## 2. Theory

The present study represents a high-aspect-ratio wing as a beam. A geometrically exact, mixed variational formulation is used for the structural dynamics of the wing (Hodges, 1990). For the aerodynamic loading, earlier work by the authors used two-dimensional (2-D) unsteady finite state aerodynamics theory (Patil et al., 2001a), which is an ideal approximation for high-aspect-ratio, straight wings. However, the vortex structure of the curved wing may necessitate 3-D modelling for accurate prediction of loads. Here a nonplanar doublet-lattice method (Albano and Rodden, 1969) is used to calculate the unsteady aerodynamic loads on the curved wing. This theory is a geometrically exact approximation of fixed-wake unsteady aerodynamics.

### 2.1. Nonlinear structural theory

The structural formulation used in the present research is based on the mixed variational formulation for dynamics of moving beams developed by Hodges (1990). Equations of motion are generated by including the appropriate energies in the variational principle followed by application of calculus of variations and spatial discretization. The formulation is described in detail in Patil (1999). The final equations are presented here for the sake of completeness.

By using simple shape functions, the mixed variational formulation leads to a set of coupled nonlinear differential equations in terms of the element nodal displacements ($u$) and rotations ($\theta$), nodal internal forces ($F$) and moments ($M$), and linear and angular velocities ($V$ and $\Omega$). Rodrigues parameters are used as rotational variables ($\theta$). The equations presented below are all matrix equations for each finite element in the structural representation. The equations have been simplified to the greatest extent possible. Since we are only considering a cantilevered wing rather than a free aircraft, we may exclude any nonlinear terms arising from the kinetic energy, because the only problems that are of interest in the present work are nonlinear static equilibrium and dynamic small perturbation analysis. The equations for the $n$th element can thus be written as

$$
\begin{aligned}
&\frac{F^{n+1}-F^n}{\Delta\ell}+f_{\text{aero}}+\widetilde{\overline{\kappa}}^n\left(\frac{F^n+F^{n+1}}{2}\right)-\dot{\overline{P}}^n=0,\\
&\frac{M^{n+1}-M^n}{\Delta\ell}+m_{\text{aero}}+(\widetilde{e}_1+\widetilde{\overline{\gamma}}^n)\left(\frac{F^n+F^{n+1}}{2}\right)+\widetilde{\overline{\kappa}}^n\left(\frac{M^n+M^{n+1}}{2}\right)-\dot{\overline{H}}^n=0,\\
&\frac{u^{n+1}-u^n}{\Delta\ell}+e_1-\overline{C}^{is}(\overline{\gamma}^n+e_1)=0,\\
&\frac{\theta^{n+1}-\theta^n}{\Delta\ell}-\left(I+\frac{\widetilde{\overline{\theta}}^n}{2}+\frac{\overline{\theta}^n(\overline{\theta}^n)^{\mathrm{T}}}{4}\right)\overline{\kappa}^n=0,\\
&\dot{u}^n-C^{is}V^n=0,\\
&\dot{\theta}^n-\left(I+\frac{\widetilde{\theta}^n}{2}+\frac{\theta^n(\theta^n)^{\mathrm{T}}}{4}\right)\Omega^n=0,
\end{aligned}
\tag{1}
$$

where $\overline{\gamma}$ and $\overline{\kappa}$, the strains and curvatures, are related to $F$ and $M$ via a cross-sectional constitutive law, and, $\overline{P}$ and $\overline{H}$, the linear and angular momenta, are related to $V$ and $\Omega$ using the cross-sectional inertial properties as

$$
\begin{aligned}
&\begin{Bmatrix}\overline{\gamma}^n\\ \overline{\kappa}^n\end{Bmatrix}=[\boldsymbol{S}]\begin{Bmatrix}\frac{1}{2}(F^n+F^{n+1})\\ \frac{1}{2}(M^n+M^{n+1})\end{Bmatrix},\\
&\begin{Bmatrix}\overline{P}^n\\ \overline{H}^n\end{Bmatrix}=[\boldsymbol{I}]\begin{Bmatrix}\frac{1}{2}(V^n+V^{n+1})\\ \frac{1}{2}(\Omega^n+\Omega^{n+1})\end{Bmatrix}.
\end{aligned}
\tag{2}
$$

The direction cosine matrix $C^{is}$ transforms the components of a vector from the $s$-frame corresponding to the deformed structure to $i$-frame corresponding to the undeformed straight wing. The direction cosine can be written in terms of the Rodrigues parameters as

$$
C^{is}=\frac{(1-\frac{1}{4}\theta^{\mathrm{T}}\theta)\Delta+\tilde{\theta}+\frac{1}{2}\theta\theta^{\mathrm{T}}}{1+\frac{1}{4}\theta^{\mathrm{T}}\theta},
\tag{3}
$$

where $\Delta$ is the 3×3 identity matrix. Finally, $f_{\text{aero}}$ and $m_{\text{aero}}$ are the aerodynamic forces and moments acting on each element, and are calculated using geometrically exact aerodynamic theory described in the following section.

### 2.2. Aerodynamic theory

It is the aim of the present work to study the complete unsteady aerodynamic loads on a nonplanar wing. The nonplanar, doublet-lattice theory (Albano and Rodden, 1969; Stahl et al., 1968; Kalman et al., 1969) is ideal for the present work. The doublet-lattice method is an unsteady, fixed-wake, lifting surface theory. It can account for geometrically exact placement of the bound and wake vortices. On the other hand, the analysis is valid for only small dynamic deformations about the large steady state. It is thus valid for the present analysis which includes nonlinear (large deformation) steady state analysis followed by linearized (small deformation) dynamic analysis. Since the wake is based on a fixed-wake model, some higher-order wake motion effects are not captured. Approximate solutions are obtained by discretizing the surface into a set of lifting elements which are short line segments of acceleration–potential doublets. The normal velocity induced at any point due to a lifting element is given by the integral of a subsonic kernel. The amplitude of the lifting elements are determined by satisfying normal velocity boundary condition at various collocation points.

It is known that the doublet-lattice results do not converge well for the quasi-steady part. Thus, nonplanar vortex-lattice method is used to calculate the quasi-steady part, while doublet-lattice calculations are used only for the incremental unsteady portion.

Once the wing is discretized and divided into trapezoidal panels (Albano and Rodden, 1969), the aerodynamic influence coefficients (AICs) are calculated. The AICs give the normal velocity induced at the collocation points due to the lifting elements. The normal downwash-pressure relationship can be written in terms of the AICs as

$$\{w\} = [D(X, k, M_\infty)]\{p\}, \tag{4}$$

where $\{w\}$ are the nondimensional normal velocities at the collocation points, $\{p\}$ are the nondimensional pressures induced due to the lifting elements and $[D]$ is the matrix of AICs. The AICs are functions of the reduced frequency ($k = \omega b/V$), freestream Mach number ($M_\infty$) and the aerodynamic surface geometry (calculated from the structural variables $X$). Since the AICs are functions of the structural deformation, the above aerodynamic relationship is nonlinear. The AICs are calculated by linearizing about a given steady state, and used in the solution procedure described in the next section. The AICs can be split into two parts, one due to the vortex distribution denoted by $[D_s^{\text{vort}}]$ for the quasi-steady part and one due to the incremental doublet distribution denoted by $[\Delta D^{\text{doub}}]$ for the unsteady part.

The forces and moments on the wing ($F_{\text{aero}}$) can be calculated by integrating the pressures over the chordwise boxes as

$$\{F_{\text{aero}}\} = q[B]\{p\}, \tag{5}$$

where $q$ is the free-stream dynamic pressure, and $[B]$ is the matrix that relates the box pressures to the sectional lift and moment.

The downwash at various collocation points on the wing can be written explicitly in terms of the angle of attack of the undeformed wing and the structural deformations and velocities. Representing the structural variables by $X$ we have

$$w = W(X) + w_0, \tag{6}$$

where $[W(X)]$ is the nonlinear transformation from the structural variables to the downwash at collocation points, and $W_\phi$ is the downwash at collocation points due to the angle of attack of the undeformed wing.

Thus, the aerodynamic lift and moment at various spanwise locations can be written in terms of the control point deflections as

$$\{F_{\text{aero}}\} = q[B]([D_s^{\text{vort}}(X, M_\infty)] + [\Delta D^{\text{doub}}(X, k, M_\infty)])^{-1}\{W(X) + w_0\}. \tag{7}$$

### 2.3. Nonlinear aeroelastic analysis

The nonlinear aeroelastic equations are obtained by substituting the expression for the aerodynamic load (Eq. (7)) into the structural equations (Eq. (1)). For simplicity, the nonlinear aeroelastic equations of motion are written in terms of the structural ($F_S$) and loading ($F_L$) operators as

$$F_S(X, \dot{X}) - F_L(X, k, M_\infty, w_0) = 0. \tag{8}$$

The solutions of interest for the above equation can be expressed in the form

$$\{X\} = \{\overline{X}\} + \{\widehat{X}(t)\}, \tag{9}$$

where $\overline{()}$ denotes the steady state value and $\widehat{()}$ denotes the small perturbation about the steady state.

For the steady state solution ($\overline{X}$) one has to solve a set of nonlinear equations given by

$$F_S(\overline{X},0)+F_L(\overline{X},0,M_\infty,w_0)=0. \tag{10}$$

The steady state solution can be found very efficiently using the Newton–Raphson method. The Newton–Raphson method calculates the solution of the nonlinear problem iteratively by solving a linearized problem at each step. The linearized problem is written in terms of the Jacobian matrix of the system as

$$\left[\frac{\partial F_S}{\partial X}\right]_{X=\overline{X}^i}\{\overline{X}^{i+1}-\overline{X}^i\}+\left[\frac{\partial F_L}{\partial X}\right]_{X=\overline{X}^i}\{\overline{X}^{i+1}-\overline{X}^i\}+F_S(\overline{X}^i,0)+F_L(\overline{X}^i,0,M_\infty,w_0)=\{0\}. \tag{11}$$

The structural Jacobian matrix of the above set of nonlinear equations can be obtained analytically and thus the nonlinear steady state solution is computationally efficient. The aerodynamic Jacobian matrix can be written using Eq. (7) as

$$\left[\frac{\partial F_L}{\partial X}\right]_{X=\overline{X}^i}=q[B]([D_s^{\text{vort}}(\overline{X}^i,M_\infty)])^{-1}\left[\frac{\mathrm{d}W}{\mathrm{d}X}\right]_{X=\bar{X}^i}. \tag{12}$$

For calculating the flutter solution, the system matrix is dynamically linearized about the steady state. Thus we get

$$\left[\frac{\partial F_S}{\partial \dot{X}}\right]_{X=\overline{X}}\{\dot{\widehat{X}}\}+\left[\frac{\partial F_S}{\partial X}+\frac{\partial F_L}{\partial X}\right]_{X=\overline{X}}\{\widehat{X}\}=\{0\}. \tag{13}$$

The structural Jacobian matrix is again derived analytically, and the aerodynamic Jacobian matrix is a function of the aerodynamic influence coefficients ($[D_s^{\text{vort}}+\Delta D^{\text{doub}}]$) determined for the calculated equilibrium wing shape.

The form of the dynamic aeroelastic equation given above is not amenable to flutter calculations due to the dependence of the aerodynamic part on the reduced frequency ($k$) as well as the velocity. To use the $k$ method (also known as the $V$–$g$ method) for flutter calculations, the equation needs to be converted to a form that is only a function of $k$. For such a transformation, the structural equations should be in second-order form. Such a transformation of the structural system is possible for the present structural model. The following steps are required in the transformation:

(i) represent the nodal forces and moments in terms of the nodal displacements and rotations using the strain–displacement equations along with the constitutive law;
(ii) represent the nodal velocities (linear and angular) in terms of the time derivatives of the nodal displacements and rotations using the displacement–velocity relations; and
(iii) substitute the calculated relations in the equations of motion to generate equations of motion only in terms of the displacements and rotations and its corresponding time derivatives.

It should be noted that the above transformations are conducted on linearized equations and thus essentially involve solving a linear system of equations.

With the above transformations we have the equations of motion as

$$[M]\{\ddot{x}\}+[K]\{x\}+q[A(k,M_\infty)]\{x\}=\{0\}, \tag{14}$$

where $x$ is the column matrix containing nodal values of displacement and rotation variables; all other structural variables in $X$ have been eliminated. Now, by assuming the motion of the form $\mathrm{e}^{\mathrm{i}\omega t}$ the above equation can be converted to a form that can be used for flutter calculation by $k$ method. The eigenvalue problem is posed in terms of the reduced frequency as

$$\frac{1+\mathrm{i}g}{\omega^2}[K]\{x\}=\left([M]-\frac{\rho b^2}{2k^2}[A(k,M_\infty)]\right)\{x\}. \tag{15}$$

The above equation can be solved using the $k$ method to obtain the flutter speed.

## 3. Results and discussion

The objective of the present investigation is to understand the effect of large deformations on the aeroelastic behavior of high-aspect-ratio wings. The effort is also directed towards investigating the nature and occurrences of these effects as well as necessary and sufficient features of theories that are to be used in predicting them. Results are obtained with increasing complexity so as delineate the importance of various nonlinear effects. The results presented herein begin with lift distribution on a rigid wing and end with the effect of deformation on flutter speed. The case used for this study is a slender wing with semispan aspect-ratio of 16 (Patil et al., 2001a). The structural properties of the wing are given in

Table 1. The results presented use a $16 \times 1$ grid for steady state calculations carried out with the vortex-lattice method and a grid of $48 \times 6$ for the unsteady calculations using a combination of vortices and doublets.

### *3.1. Nonlinear steady state results*

The first case to be studied is that of a curved but rigid wing. The curved wing model is generated by applying a distributed load on the flexible straight wing so as to get a tip displacement of 4 m (25% of the span). The aim is to investigate the effect of the curvature on the 3-D aerodynamic load characteristics. Fig. 1 shows the lift distribution on a curved wing normalized using the 2-D lift. Lift distributions obtained by using nonplanar geometry as well as those assuming planar geometry are presented. For the case of a constant distribution of angle of attack, it is seen that the planar aerodynamic theory predicts the aerodynamic loads very accurately. Thus, the aerodynamic nonlinearity due to nonplanarness seems to be quite negligible for the static loads, even for the large deformations considered here. As expected, the 3-D effects are also quite small and change only the lift distribution near the tip.

Also shown in Fig. 1 are lift distributions obtained if one assumes an applied root angle of attack. Due to the curvature of the wing, there is a decrease in the exact angle of attack (the angle between the airflow and the deformed surface) with deformation. Such a decrease in the angle of attack leads to decrease in the lift. This is indicated by lower lift over the span, as shown in the figure. The largest decrease is near the tip since the deformation and thus the decrease in the angle of attack is largest at the tip.

Table 1
Model data

| | |
|---|---|
| Half-span | 16 m |
| Chord | 1 m |
| Mass per unit length | 0.75 kg/m |
| Moment of inertia (50% chord) | 0.1 kg m |
| Spanwise elastic axis | 50% chord |
| Center of gravity | 50% chord |
| Bending rigidity | $2 \times 10^4$ N m$^2$ |
| Torsional rigidity | $1 \times 10^4$ N m$^2$ |
| Bending rigidity (edgewise) | $5 \times 10^6$ N m$^2$ |
| Flight condition | |
| Altitude | 20 km |
| Density of air | 0.0889 kg/m$^3$ |

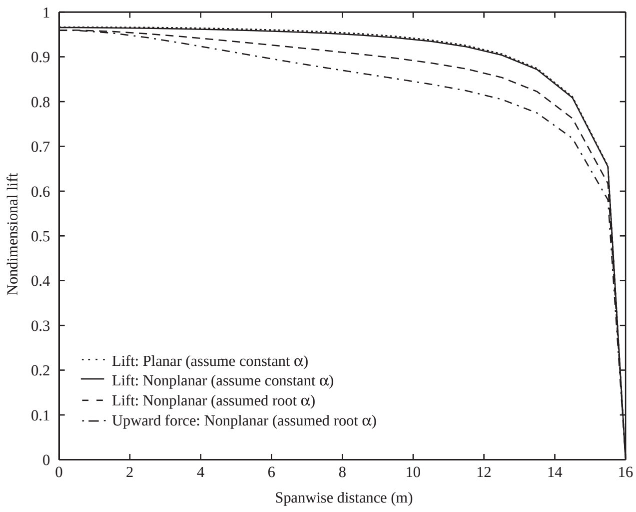


Fig. 1. The effect of geometrical nonlinearities on the lift generated by a curved rigid wing.

Finally, for complete geometrical exactness the aerodynamic force is applied perpendicular to the wing reference line and the flow direction. Thus, there is a decrease in the lift component in the upward direction. This is indicated on another curve in Fig. 1. The upward force results are again lower than the lift results due to the deformation. Thus, one finds that the effect of nonplanar geometry on the airloads (due to nonplanar bound and wake vortex distribution) is negligible, while a consistent modelling of the interface (transfer of angle of attack information from the structural model to the aerodynamic model and the transfer of load information from aerodynamic model to the structural model) is significant.

The second study involves the flexible wing flying with a speed of 25 m/s at an angle of attack of 10°. Fig. 2 shows the steady state equilibrium solution obtained using a nonlinear theory and compares it to that obtained using a simple linear theory. It is seen that the bending deformation is predicted quite accurately by the linear theory except for the fore-shortening effect. Similarly, the wing twist is predicted quite accurately using the linear theory. Finally, the lift distribution predicted by the planar aerodynamic theory is also quite close to the complete nonplanar theory. The difference can be attributed mostly to the decrease in angle of attack due to bending. The nonplanar aerodynamics does not play an important part, nor does the structural nonlinearity, even for wing deflections of the order of 25% of the span.

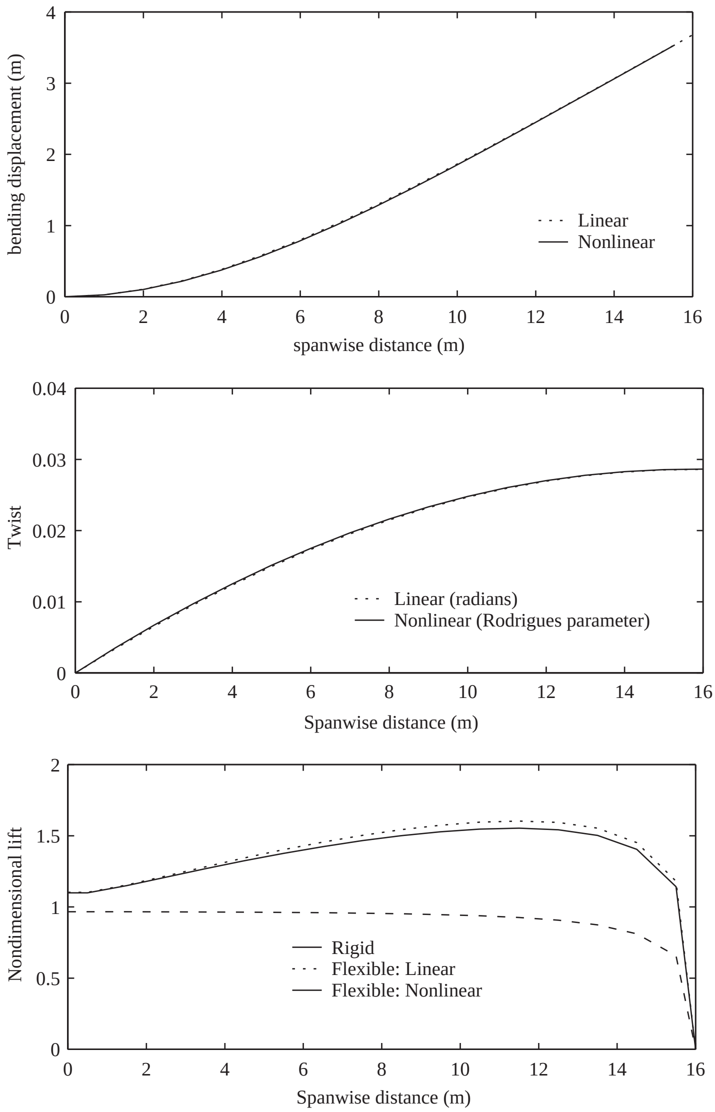


Fig. 2. Comparison between linear and nonlinear equilibrium for the wing at a speed of 25 m/s and angle of attack of 2°.

### 3.2. Linearized dynamic results

To investigate the changes in the dynamic properties of the aeroelastic system, nonlinear aeroelastic analysis is conducted on the wing model about a steady state obtained by assuming a constant load distribution over the wing. Here it should be mentioned that one could instead obtain a steady state by assuming a wing angle of attack. But matching the flutter speed using the $k$ method becomes quite tedious. Since it is the aim of the present study to investigate the changes in the dynamics of the system for various steady states (or loading conditions), such a study is simplified by using a constant distributed loading to mimic the aerodynamic loading.

The first investigation of the dynamic behavior involves the study of the oscillatory loads on a curved wing using a planar and nonplanar analysis. Fig. 3 shows the complex oscillatory lift predicted for a wing curved to around 25% of the span and oscillating in the first wing bending mode with a reduced frequency $k = 0.4$. In a manner similar to that exhibited by the steady-lift results, the unsteady lift also shows a negligible effect of the nonplanar geometry.

The second study involves the calculation of the structural dynamic properties of the wing at various loading conditions. The maximum loading leads to a wing tip deflection of around 25% of the span. Fig. 4 shows the change in the structural dynamic frequencies with wing loading. The flatwise frequencies seem to remain almost constant. The edgewise bending and torsion modes are the ones that are significantly affected. The deflection of the wing leads to a structural coupling between the torsion and edgewise bending modes. For the case considered, there is a 70% decrease in the frequency of the mode that is purely torsional in the unloaded state over the full range of loading considered. Also, there is a 25% increase in the frequency of the mode that is purely edgewise bending in the unloaded state over that same range. These changes in frequency are quite significant. Thus, it is very important to model the nonlinear structural dynamic characteristics accurately.

Though change in frequencies is a matter of concern, it is the change in aeroelastic characteristics that is most important. Fig. 5 shows the changes in the flutter speed and frequency with wing loading. There is a significant decrease in flutter speed with increase in wing loading. This can be attributed to the decrease in the torsional frequency with wing loading. There is a decrease of almost 50% in flutter speed, which significantly compromises the flight envelope. Therefore, it is necessary to include such effects—not only during the detailed analysis stage, but even during the preliminary design stage, the stage at which the avoidance of such deleterious effects could effect the largest cost reduction.

The critical changes in the aeroelastic characteristics of the wing can be attributed to the dominant effect of the nonlinear steady state on the structural dynamic characteristics of the wing. The aeroelastic solution obtained using a nonlinear structural model coupled with a 2-D aerodynamics model is also shown in Fig. 5. Such a model predicts the trend in the nonlinear behavior of the wing since it captures the dominant structural nonlinearity as well as the interface nonlinearities.

The 3-D flutter results can be improved upon by using a finer grid. Table 2 shows the convergence of the flutter speed with grid size. It is seen that a high number of boxes are required for the high-aspect-ratio wing considered here since

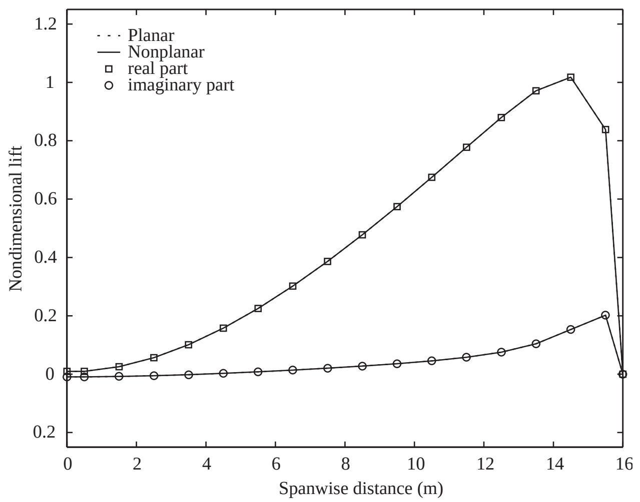


Fig. 3. The effect of geometrical nonlinearities on the lift generated by a curved wing vibrating in its first flatwise bending mode at a reduced frequency of 0.4.

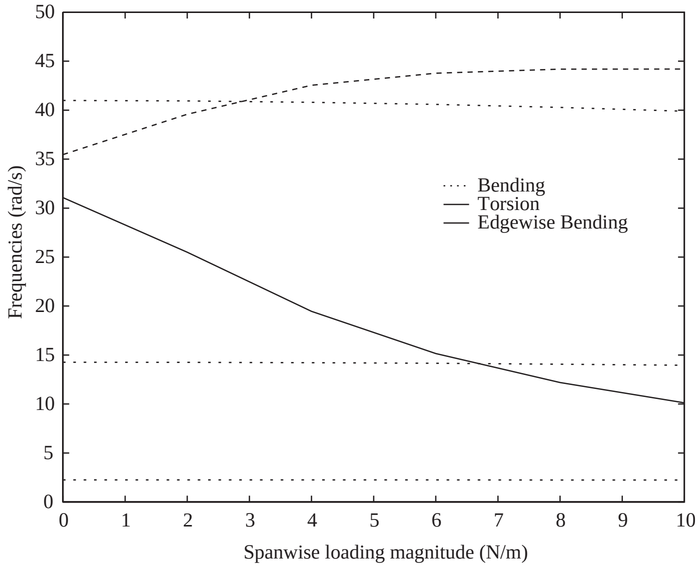


Fig. 4. The effect of geometrical nonlinearities on the structural frequencies.

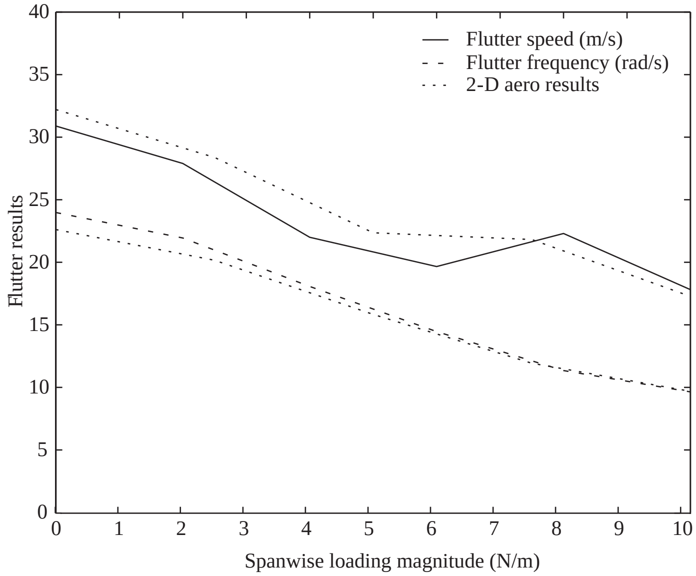


Fig. 5. The effect of geometrical nonlinearities on flutter.

Table 2
Convergence of flutter results

| Theory | Flutter speed | Flutter frequency |
|---|---|---|
| 2-D (Peters) | 32.21 | 22.61 |
| 3-D Nonplanar (doublet + vortex) | | |
| Grid: $32 \times 4$ | 29.64 | 24.82 |
| Grid: $48 \times 6$ | 31.01 | 24.04 |
| Grid: $64 \times 8$ | 31.57 | 23.69 |
| Grid: $128 \times 8$ | 31.75 | 23.60 |

the box aspect-ratio must be near unity for accurate airload prediction. In this respect, a modified doublet-lattice algorithm that is accurate for higher box aspect-ratios will be useful.

## 4. Conclusion

A nonlinear aeroelastic analysis has been presented. The aeroelastic analysis uses a geometrically exact structural theory and a nonplanar, fixed-wake aerodynamic theory. The aeroelastic analysis methodology also consistently models the fluid–structure interface.

The results presented shed light on the importance of various types of geometrical nonlinearities on the aeroelastic behavior of high-aspect-ratio wings. It is seen that there is negligible difference between the airloads calculated using the correct nonplanar wing geometry as compared to loads calculated assuming a planar wing. The nonplanar modelling of the wing is not essential even for high wing deflection. In fact, even the 3-D effects are quite small for the high-aspect-ratio wing considered. More research using a free-wake aerodynamic model is required to accurately judge the 3-D nonplanar aerodynamic effects, especially for large amplitude oscillations.

The structural nonlinear effects were quite small for the steady state calculations. On the other hand, the dynamic properties of the structure undergo significant change with wing deformation. The curved wing structural dynamic frequencies are quite different from those of the straight wing. The dependence of the structural dynamic properties on the wing deformation is the dominant nonlinearity for long slender wings. Such changes in the frequencies and modeshapes of the system leads to a significant change in the dynamic aeroelastic behavior of the wing. The flutter speed of the system can change by more than 50% for large deflections.

Finally, it should also be noted that the geometrically exact calculation of the angle of attack and proper application of the airloads are also necessary for correct prediction of aeroelastic response. These effects were termed as interface nonlinearities in the present paper. Some researchers may instead group parts of these nonlinearities in the structural and/or aerodynamic models, possibly leading to conclusions different from those given above.

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